SkillsTutor™
Math Fact Fluency

Research Basis

HOUGHTON MIFFLIN HARCOURT

SkillsTutor MATH FACT FLUENCY

Research Basis

Introduction

"Practice your facts."

It's a statement spoken by math teachers to students virtually every day. When a parent asks how he can help his child succeed in math, a common response from the teacher is to practice math facts daily. For students, practicing math facts has meant taking timed tests of 50 to 100 facts, each time hoping to better your score from the last test. For parents, practicing math facts with their children meant flipping through a stack of flash cards while their children speedily answered some and used their fingers to solve others. *SkillsTutor Math Fact Fluency* provides teachers and parents with a new solution to math fact practice.

What is math fact fluency?

Math fact fluency is the ability to recall the answers to basic math facts automatically and without hesitation. Fact fluency is gained through significant practice, with mastery of basic math facts being a goal of both teachers and parents. First, though, it is very important that all students understand the concepts of addition, subtraction, multiplication and division. In what is probably the most common method of teaching addition, students combine sets of objects to find the total number of objects. Students form equal groups of objects to understand multiplication and separate a group of objects into equal groups to understand division. Through either direct instruction or discovery, students learn the commutative properties of both addition and multiplication.

Once a student understands the concept of a particular operation, he can begin to practice solving facts under un-timed conditions. Students generally employ a number of strategies to solve basic facts. Common

addition strategies include adding 1 or 2 (counting on), making 10, doubles and doubles + 1. Repeated addition and skip counting are examples of multiplication strategies.

Spear-Swerling (2006) states that both conceptual understanding and the ability to solve facts accurately under un-timed conditions are prerequisites for automatic recall of facts. When these prerequisites have been achieved, it is imperative that the student practices the facts such that they can be answered automatically (*i.e.,* retrieval of the answer is both quick and accurate). Through repeated exposures with a fact, the brain establishes a memory relationship with the fact, leading to automatic retrieval. The usual speed goal for automatic recall of facts when responding orally is 30 correct per minute—2 seconds per fact.

Learning and memory

The rationale for the need to memorize facts is derived from the "information processing" model of the human brain. It's familiar to most people in the concepts of short-term, or working, memory and long-term memory. Research, as noted by Willingham (2004), clearly shows that to commit information to both short-term and long-term memory, ongoing practice is necessary.

Short-term memory has limited storage capacity, with material being retained only as long as it is being consciously practiced. Long-term memory is intended to store information for a long period of time. A study by Semb, Ellis & Araujo (1993) showed that if content is studied for four months or eleven months, it will be well retained for about one year after the last practice, with most of the content forgotten by end of three or four years without further practice. Bahrick (1984) and Bahrick & Hall (1991) found that if material is studied for three or four years, the information may be retained for as long as 50 years after the last practice. The key component in the retention of information for that long a period of time is the additional practice completed by the student.

Within long-term memory, there are two memory systems: one concerned with declarative knowledge and the other with procedural knowledge. Declarative knowledge is concerned with "knowing that," and procedural knowledge is concerned with "knowing how." If a student if asked, "What is 8 x 7?", the difference lies between responding immediately with "56," (declarative), or thinking "8 groups of 7 items" (procedural). When a student is able to quickly and accurately retrieve the answer to a math fact, the student has a strong declarative knowledge system for that fact.

With practice, solving math facts is a process that moves from procedural to declarative understanding. Using the addition fact 2 + 4, Garnett (1992) shows how the strategies for solving addition facts develop over time. The student moves from a procedural understanding, in which he counts each set, to declarative knowledge, demonstrating the ability to respond immediately with the correct sum.

Strategy	Representative use to solve 2 + 4
Counting all	"1, 2 . . . 1, 2, 3, 4 . . . 1, 2, 3, 4, 5, 6"
Short-cut sum	"1, 2, 3, 4, 5, 6"
Finger display	Displays 2 fingers, then 4 fingers, says "6"
Counting-on-from-the-first-addend	"2 . . . 3, 4, 5, 6" or "3, 4, 5, 6"
Counting-on-from-the-larger-addend	"4 . . . 5, 6" or "5, 6"
Linking	"2 + 2 = 4, + 2 more = 6"
Retrieval	"6"

Research by Cumming and Elkins (1999) highlights a common misconception about math fact strategies: natural development or direct instruction of strategies will result in math fact automaticity. Many students, most notably students with learning disabilities, fail to naturally develop

fact strategies. Furthermore, those students that do not move beyond strategies to an automatic retrieval of facts continue to expend some mental effort to solve facts—effort that could be applied to solving higher-level mathematical problems.

Why is math fact fluency important?

Information processing theory supports the view that automaticity in math facts is fundamental to success in many areas of higher mathematics. Without the ability to retrieve facts directly or automatically, students are likely to experience a high cognitive load as they perform a range of complex tasks. The added processing demands resulting from inefficient methods such as counting (vs. direct retrieval) often lead to declarative and procedural errors (Woodward, 2006; Cummings & Elkins, 1999; Pellegrino & Goldman, 1987; Hasselbring, Goin, & Bransford, 1988).

As students progress through their formal schooling, the math concepts they learn often build upon one another. For instance, in 2nd and 3rd grade, students learn perimeter by adding the lengths of the sides of a shape. In 4th grade, students use formulas to find the perimeters of rectangles, squares and other objects that have sides measuring the same lengths. These formulas require the knowledge of multiplication facts. By 5th grade, the lengths of the sides have increased to 2-digit numbers, requiring proficiency in multi-column addition or multiplication. In order to solve for the perimeter of a shape, the student needs to understand the process of finding the perimeter. The student then needs to use his computation skills to solve for the correct answer. Imagine the processing demands required to complete these tasks for a student who has failed to master his basic facts.

Accurate and efficient retrieval of basic math facts is critical to a student's success in mathematics. Research published by The National Council of Teachers of Mathematics (NCTM) states that students who cannot retrieve basic facts easily get lost and often cannot follow the logic of an explanation given by the teacher or a peer when the problems are embedded within more complex mathematic operations, such as simple algebra or long division. The teacher or the textbook assumes virtually automatic retrieval of these facts and bases explanations on this assumption.

NCTM, The National Math Panel and *Math Fact Fluency*

NCTM published its *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* in 2006, which outlines three key skills each student needs to master at each grade level. In grades 1 through 4, the *Focal Points* define critical skills in all four basic operations, including the ability to instantly recall those basic facts.

In April 2006, the National Math Panel was created by the President and charged to recommend ways "...to foster greater knowledge of and improved performance in mathematics among American students." Supporting much of the research that has existed regarding automaticity of basic facts, the Panel's report in March 2008 made it very clear that automatic recall of facts was paramount to student success in mathematics:

> A focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics, should become the norm in elementary and middle school mathematics curricula. Any approach that continually revisits topics year after year without closure is to be avoided...By the term proficiency, the Panel means that students should understand key concepts, achieve automaticity as appropriate (e.g., with addition and related subtraction facts), develop flexible, accurate, and automatic execution of the standard algorithms, and use these competencies to solve problems...
>
> Computational proficiency with whole number operations is dependent on sufficient and appropriate practice to develop automatic recall of addition and related subtraction facts, and of multiplication and related division facts. It also requires fluency with the standard algorithms for

addition, subtraction, multiplication, and division. It also requires a solid understanding of core concepts, such as the commutative, distributive, and associative properties. Although the learning of concepts and algorithms reinforce one another, each is also somewhat dependent on different types of experiences, including practice.

What is SkillsTutor *Math Fact Fluency*?

Skills Tutor Math Fact Fluency is an online instructional and practice product that helps students achieve automaticity in the basic facts of addition, subtraction, multiplication and division. It is modern, personalized and fully adaptive, responding to each student's actual performance and giving practice where it will help most.

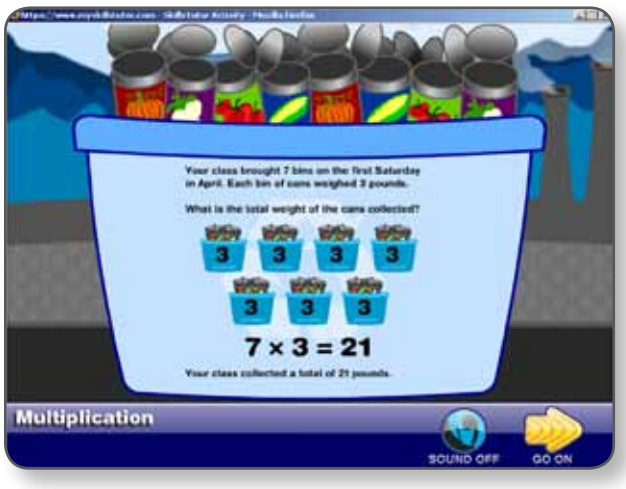
Math Fact Fluency has a scope and sequence designed to introduce new facts gradually to students as they work toward mastery. Facts are grouped in units, with each lesson in a unit introducing two new facts. Units are sequenced based on a progression of easier facts (i.e., multiplying by 2) to those generally accepted as more difficult to master (i.e., multiplying by 12). At the end of each operation, a review lesson allows students to continue practicing their facts after completion of the scope and sequence.

Supporting the instructional units are a variety of assessments designed to give teachers a thorough understanding of each student's strengths and weaknesses. Teachers can assign a placement test in each operation to assess students' existing fact mastery and place each student into the 1st unit where mastery has not been demonstrated. Benchmark tests are designed to provide teachers with a monthly window into a student's fact mastery using a 100-question assessment, similar to a timed test given on paper. A final test shows the overall progress at the end of an operation.

During each student's first experience with the product, the student is required to create a character of himself. This fun and exciting tool allows the student to choose their facial features, as well as their clothing and accessories. The character that each student creates is then featured in each lesson throughout the product. Klem and Connell's research on engaging youth in school acknowledges a consensus in research that personalized learning environments engage students in their schoolwork. Improved academic performance is one benefit reaped by students who are engaged in their schoolwork.

Math Fact Fluency's default fact mastery time can be adjusted upwards and downwards for each student. Additionally, a student can be set to use an "untimed mode." This feature is especially important for special needs students, who might be slow to interact with the computer, but who would still benefit from intensive practice and positive reinforcement.

Each *Math Fact Fluency* lesson uses an authentic, "real-world" scenario in which a student would actually use math. Salend (1994) recommended that new math concepts be introduced through everyday situations as opposed to worksheets. With everyday situations as motivators, students are more likely to recognize the importance and relevance of a concept. In *Math Fact Fluency,* students will find themselves learning the focus facts of a lesson in authentic situations such as working at a school bake sale, tracking minutes in a reading log, and making multiple batches of a recipe.

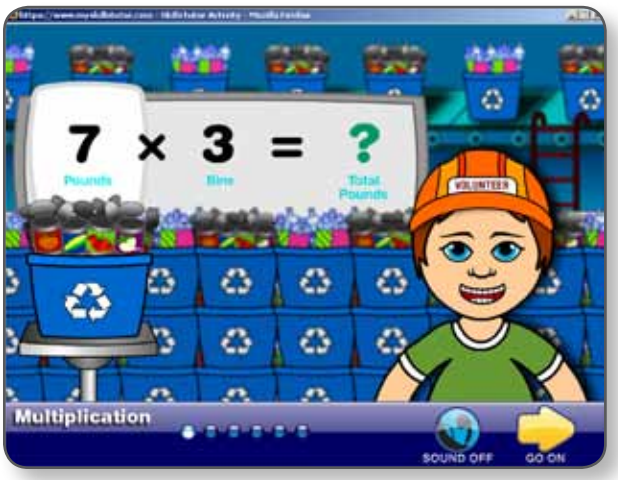


Sample tutorial screen

These real-life situations, along with more than 35 other scenarios, reinforce to students that math is an important part of the activities in their daily lives. The combination of personalizing the instruction with a self-created character and real-world scenarios provides students with a strong motivation to succeed. Carpenter, Fennema, Peterson, Chiang & Loef (1989) further support that contextualizing instruction can make seemingly mundane computation exercises fascinating to children.

Following each lesson's tutorial is the student's opportunity to practice the lesson's facts, as well as facts taught previously to the student. Again, the focus of *Skills Tutor Math Fact Fluency* is to provide regular, focused practice. In a study conducted by Brookhart, Andolina, Zuza and Furman (2004), students reflected on their progress of learning their multiplication facts and completed self-assessments of their performance. The main reason given by students whose performances did not meet expectations was "didn't practice enough."

After the lesson's two focus facts are introduced, the student is given practice on these facts only. In addition and multiplication, this practice is followed with the commutative facts of the lesson's two focus facts. Thus, if the student is learning 5 x 10, he will be given practice with the related fact 10 x 5. By relating a fact currently being learned with a previously learned fact, the brain's ability to retain the facts is strengthened.



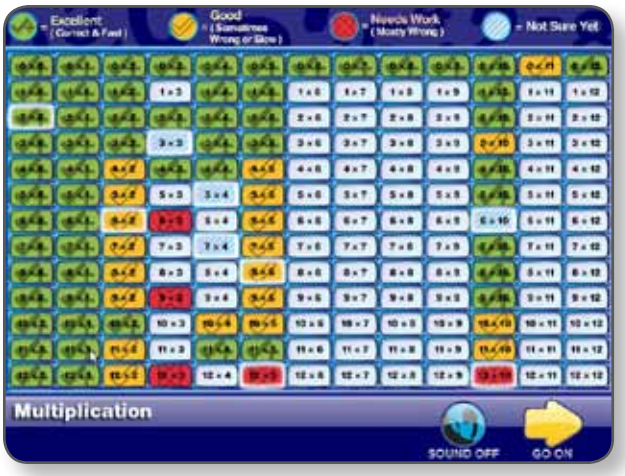
Sample practice screen with student avatar

Research shows that there is a significant memory benefit to distributing practice over several sessions. This phenomenon is known as the spacing effect and was identified in the late 1800s by Ebbinghaus, a pioneer in the study of memory. The spacing effect was tested using lists of meaningless syllables, but has since been applied to other types of material, including the learning of math facts. Rea and Modigliani (1985) conducted an experiment with 3rd graders which showed that students taught facts in a distributed condition outperformed the others in the study.

Using an analytical and adaptable algorithm, two additional groupings of facts provide students with multiple exposures to practice facts learned in past lessons. First, facts that the student has learned in his past two lessons are added to the current lesson's focus facts. This spiraling provides the practice necessary to strengthen the student's memory of the facts. The selection of a final set of nine facts for practice is heavily influenced by the speed of prior responses on facts, how often the student has seen particular facts and whether more data is needed to consider facts mastered or not. It is these final nine facts that provide the student with the most individualized practice as their selection is based on the student's prior performance. Thus, students who are taking the same lesson path through *Math Fact Fluency* will have unique experiences based on their speed of retrieval and accuracy of answers.

Willingham (2004) stresses that "it is difficult to overstate the value of practice. For a new skill to become automatic or for new knowledge to become long-lasting, sustained practice, *beyond the point of mastery,* is necessary…Studying material that is already known is called *overlearning*. Because memory is prone to forgetting, one cannot learn material to a criterion and then expect the memory to stay at that level very long." By reinforcing the facts previously learned by the student through ongoing practice, *Math Fact Fluency* is continually moving the student toward automaticity.

After the student completes his fact practice, he will receive his lesson score. While the lesson score is based on the student's progress with the lesson's two focus facts, it is followed by a fact matrix, which is a graphical view of how he is progressing through his facts.



Sample fact matrix

A study completed by Wood and Frank (2000) required students to graph their progress in learning their multiplication facts. Their research showed that most students are highly motivated when they observe the progress they are making and strive to reach the goals shown on their graphs. On the fact matrix, green (✓+) means the fact is considered mastered. Red means it is a problematic fact, and the student has given multiple wrong answers. Yellow (✓) indicates mixed performance, and white indicates there is not enough data to make a judgment. Fact mastery only occurs over multiple lessons.

The data retrieved from each student's lessons is stored in a robust learning management system, which is used by teachers to set up assignments and monitor student activity. In addition to the reports available to all SkillsTutor users, a number of reports have been developed specifically for *Math Fact Fluency:*

- **Fact Matrix Report**
  Similar to the fact matrix displayed to the student at the end of every lesson, this report displays the red, green and yellow indicators of student mastery. There is also a slider on the bottom of the report that allows the teacher to adjust each student's performance default fact mastery time. For example, a teacher might choose to move the slider up to accept greater response times for special needs students. The teacher might also choose to move the slider down to give less time for more advanced students.

- **Calendar Report**
  Regular usage is critical to a student's success in *Math Fact Fluency.* The calendar report displays each student's daily time-on-task within the product in a clear calendar format. While designed for *Math Fact Fluency,* this report is also available to any SkillsTutor user.

- **Fact Assessment Report**
  This report gives a class-wide view of performance on any of the placement, final or monthly benchmark tests. It will report the score within each fact group for all students in the class. It helps the teacher to measure gross progress and to direct individual students or the class as a whole.

The strength of the reports is that they enable teachers to effectively and efficiently monitor the progress of their students and make changes to each student's assignments in order to best maximize student time-on-task.

Automaticity and Students with Learning Disabilities

Creating an environment where students with learning disabilities can find success in basic fact mastery is a challenge faced in all schools. More and more students with learning disabilities are receiving their mathematics instruction

in standard classrooms. In these classrooms, students are typically taught strategies as the means to solve basic math facts.

However, the natural development of strategies to solve basic facts is elusive to many learning-disabled students. Goldman, Pellegrino and Mertz (1988) cite that learning-disabled students generally solve an addition problem such as 2 + 9 by counting on from the first addend, instead of from the larger addend. Goldman and her colleagues do believe that an organized approach to practice can address the delay exhibited by these students.

Research by Hasselbring, Goin and Bransford (1988) showed a clear discrepancy between the number of facts that students with and without learning disabilities could retrieve automatically. A discrepancy existed as early as age 7. Not surprisingly, the discrepancy increased as the students aged. The response accuracy of these students was also measured. At age 7, learning disabled students were about 20% less accurate than students without learning disability. By age 9, the gap had closed to roughly 5%. This appears to verify an interference in the natural development of strategies to solve basic facts, yet does imply that automaticity can be developed in learning-disabled students through the right methods of instruction.

Research on the development of fact automaticity in learning-disabled students leaves little doubt as to the importance of extended practice on math facts. Pellegrino and Goldman (1987) concluded that extended practice leading to automaticity provides the learner with both time and resources to expend on metacognitive and higher-order activities.

As mentioned, *Math Fact Fluency* allows teachers to prescribe untimed lessons to students in which mastery of facts within a time limit would be too great a challenge. Research supports a number of other features of the product as being particularly beneficial to learning-disabled students:

- When presenting new facts, Lock (1997) says that "facts should be presented a few at a time with frequent repetition of previously memorized facts." Each *Math Fact Fluency* lesson introduces only two new facts. In the addition and multiplication units, practice with commutative facts is provided if those facts were previously learned. A complex algorithm provides practice with facts that have been mastered, as well as those facts needing more practice.
- McCoy and Prehm (1987) suggest that charts or graphs be used to show students their progress in fact automaticity. *Math Fact Fluency* ends each practice with a fact matrix which displays the student's progress on his path to complete fact mastery.
- Additionally, Lock (1997) states that games are a useful tool for providing additional practice with basic facts. At the conclusion of a student's lesson in *Math Fact Fluency,* an assortment of fun and motivating games provide practice with the lesson's facts, as well other facts not yet mastered.

While the features in *Math Fact Fluency* provide strong support for learning disabled students, it is a product that has been designed to help all students achieve basic math fact automaticity. With a solid research foundation and a strong emphasis on repeated practice, *Math Fact Fluency* provides both teachers and parents with a high-quality, modern solution to basic math fact automaticity.

References

Ashcraft, M. (1987). Children's knowledge of simple arithmetic: A developmental model and simulation. In J. Bisznz, C. Brainerd, & R. Kail (Eds.), *Formal Methods in Developmental Psychology: Progress in Cognitive Development Research* (302-338). New York: Springer-Verlag.

Bahrick, H. P. (1979). Maintenance of knowledge: Questions about memory we forgot to ask. Journal of Experimental Psychology: General, 108, 296-308.

Bahrick, H. P. (1984). Semantic memory content in permastore: Fifty years of memory for Spanish learning in school. *Journal of Experimental* Psychology: General, 113, 1-29.

Bahrick, H. P. & Hall, L. K. (1991). Lifetime maintenance of high school mathematics content. Journal of Experimental Psychology: General, 120, 20-33.

Brookhart, S., Andolina, M., Zuza, M. & Furman, R. (2004). Minute math: An action research study of student self-sssessment. *Educational Studies in Mathematics, 57,* 213-227.

Carpenter, T., Fennema, E., Peterson, P., Chiang, C. & Loef., M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal,* 26 (4), 499–531.

Cohen, N. J. & Squire, L. R. (1980). Preserved learning and retention of pattern analyzing skill in amnesia: Dissociation of knowing how and knowing that. *Science, 210,* 207–209.

Cumming, J. & Elkins, J. (1999). Lack of automaticity in the basic addition facts as a characteristic of arithmetic learning problems and instructional needs. *Mathematical Cognition, 5* (2), 149-180.

Garnett, K. (1992). Developing fluency with basic number facts: Intervention for students with learning disabilities. *Learning Disabilities Research & Practice, 7,* 210–216.

Goldman, S. R., Pellegrino, J.W., & Mertz, D.L. (1988). Extended practice of basic addition facts: Strategy changes in learning disabled students. *Cognition and Instruction, 5,* 223-265.

Hasselbring, T., Goin, L., & Bransford, J. (1988). Developing math automaticity in learning handicapped children: The role of computerized drill and practice. *Focus on Exceptional Children, 20,* 1-7.

Klem, A.M., and Connell, J.P. (in press). Engaging youth in school. In L.R. Sherrord, C. Flanagan, R. Kassimir (Eds.), *Youth Activism: An International Encyclopedia.* Westport, CT: Greenwood Publishing Company.

Lock, R. (1997). Adapting mathematics instruction in the general education classroom for students with mathematics disabilities. http://www.ldonline.org/article/5928

McCoy, E. M. & Prehm, H. J. (1987). Teaching mainstreamed students. Methods and techniques. Denver: Love Publishing Company.

Pellegrino, J. & Goldman, S. (1987). Information processing and elementary mathematics. *Journal of Learning Disabilities, 20,* 23-32, 57.

Rea, C. & Modigliani, V. (1985). The effect of expanded vs. massed practice on the retention of multiplication facts and spelling lists. *Human Learning: Journal of Practical Research & Applications, 4,* 11-18.

Salend, S. (1994). Effective mainstreaming: Creating inclusive classrooms. New York: Macmillan Publishing Company.

Semb, G., Ellis, J., & Araujo, J. (1993) Long-term memory for knowledge learned in school. *Journal of Educational Psychology, 85,* 305-316.

Siegler, R. (1988). Strategy choice procedures and the development of multiplication skill. *Journal of Experimental Psychology: General, 117* (3), 258-275.

Siegler, R. (1987). The perils of averaging data over strategies: An example from children's addition. *Journal of Experimental Psychology: General, 116* (3), 250-264.

Spear-Swerling, L. (2006). Developing Automatic Recall of Addition and Subtraction Facts. http://www.ldonline.org/spearswerling/9655

The National Council of Teachers of Mathematics. (2007). Research brief: What are the characteristics of students with learning difficulties in mathematics?

Willingham, D. (2002) Allocating student study time: Massed vs. distributed practice. *American Educator, Summer,* 37-39, 47.

Willingham, D. (2004). Practice makes perfect—But only if you practice beyond the point of perfection. *American Educator, Spring,* 31-33, 38-39.

Wood, D. & Frank, A. (2000). Using memory-enhancing strategies to learn multiplication facts. *Teaching Exceptional Children, 32* (5),78-82.

Woodward, J. (2006). Developing automaticity in multiplication facts: Integrating strategy instruction with timed practice drills. *Learning Disability Quarterly, 29,* 269-289.

The SkillsTutor Mission:

SkillsTutor provides:

- Instruction to learners that is efficient, relevant, engaging and targeted.
- Services to support our customers so that they will be successful at improving student achievement.
- Products that are high-quality, reliable, nimble and multi-platform.
- Tools to inform data-driven decisions.
- Reporting to meet state and federal accountability requirements.
- SkillsTutor aims to continually grow the business by investing in systems, products and people.

Our Vision:

"SkillsTutor is the company of choice for cloud-based, digital, personalized assessment and instruction to significantly increase student achievement. SkillsTutor employees accept responsibility for, and are passionate about, delivering the most valuable product of all…an educated, high-functioning global citizen."

SkillsTutor Houghton Mifflin Harcourt
222 Berkeley Street
Boston, MA 02116
www.skillstutor.com